Sociology 601 Math Refresher

Mathematics will be an important part of the course. I expect you to have competence in basic algebra and some familiarity with the rules of probability. Here is a review of some of the key skills you should have at the start of the course. If you find yourself a bit rusty, use this review as a guide for self-directed practice.

Single Summation

I use the Greek capital letter sigma (**Σ**) to designate summation.

$$\sum_{i=1}^{n} X_i$$

This expression tells you to add (sum) different values for the variable X, from X_1 to X_n.

$$\sum_{i=1}^{n} X_i = X_1 + X_2 + X_3 + \ldots + X_{n-1} + X_n$$

Here are several expressions that use **Σ**:

$$\left(\sum_{i=1}^{n} X_i\right)^2 = \left(X_1 + X_2 + \ldots + X_n\right)^2$$

$$\sum_{i=1}^{n} X_i^2 = X_1^2 + X_2^2 + \ldots + X_n^2$$

$$\sum_{i=1}^{n} c = c + c + \ldots + c = nc$$

$$\sum_{i=1}^{n} cX_i = cX_1 + cX_2 + \ldots + cX_n = c\sum_{i=1}^{n} X_i$$

$$\sum_{i=1}^{n}\left(X_i + Y_i\right) = \left(X_1 + Y_1\right) + \left(X_2 + Y_2\right)$$

$$+ \ldots + \left(X_n + Y_n\right) = \sum_{i=1}^{n} X_i + \sum_{i=1}^{n} Y_i$$

Practice exercise: Solve the above equations for n = 3, c = 11, $X_1 = 3$, $X_2 = -1$, $X_3 = 7$, $Y_1 = 2$, $Y_2 = 4$, and $Y_3 = 0$.

Product Notation:

I use the Greek capital letter pi (**Π**) to designate multiplication.

$$\prod_{i=1}^{n} X_i$$

This expression tells you to add (sum) different values for the variable *X*, from X_1 to X_n.

$$\prod_{i=1}^{n} X_i = X_1 * X_2 * X_3 * \ldots * X_{n-1} * X_n$$

Practice exercise: Solve the equation for n = 5, $X_1 = 3$, $X_2 = -1$, $X_3 = 2$, $X_4 = -4$, and $X_5 = 7$.

Algebraic Rules for Exponents and Logarithms

If you don't remember how to use exponents and logarithms, find a chapter in an old algebra book and review the topic. Here are some rules we will use.

$a^0 = 1$ (example: $3^0 = 1$)

$a^m * a^n = a^{m+n}$ (example: $4^2 * 4^3 = 4^{(2+3)} = 4^5 = 1024$)

$(a^m)^n = a^{m*n}$ (example: $(3^2)^2 = 3^{2*2} = 3^4 = 81$)

$a^{-n} = 1 / a^n$ (example: $10^{-3} = 1 / 10^3 = 1 / 1000 = .001$)

$a^m / a^n = a^{m - n}$ (example: $4^2 / 4^3 = 4^{(2 - 3)} = 4^{-1} = 1 / 4^1 = 1 / 4 = 0.25$)

$(ab)^n = a^n b^n$ (example: $(3*4)^2 = 3^{2*}4^2 = 9*16 = 144$)

There is a special number that often pops up: the base of the natural logarithm, or *e*.

$e = 2.7183\ldots$

e has many unusual properties, one of which is that as x gets closer and closer to zero, $(1+x)^{1/x}$ gets closer and closer to *e*. (This is actually cool, for reasons I will explain in class.)

$e^0 = 1$

$e^{0.7} = 2.72^{0.7} = 2.01$ (multiplying a number by $e^{0.7}$ doubles the number)

$e^{-0.7} = 2.72^{-0.7} = 0.497$ (multiplying a number by $e^{-0.7}$ halves the number)

When we use logarithms in this class, we will use the natural logarithms with base *e*.

$\ln (2.72) = 1$

$\ln(1) = 0$

$\ln (2.0) = 0.698$

$\ln (0.5) = -0.693$

Here are some general rules for logarithms:

$\ln A*B = \ln A + \ln B$ (example: $\ln (1.56 * 2.02) = \ln 1.56 + \ln 2.02 = .445 +.703 = 1.15$)

$\ln A / B = \ln A – \ln B$ (example: $\ln (1.56 / 2.02) = \ln 1.56 - \ln 2.02 = .445 -.703 = -.258$)
$\ln A^c = c \ln A$ (example: $\ln (1.56)^3 = 3*\ln 1.56 = 3*.445 = 1.335$)

$e^{\ln A} = A$ (example: $e^{\ln (1.56)} = e^{.445} = 1.56$)

$\ln e^A = A$ (example: $\ln (e^{1.56}) = \ln (4.76) = 1.56$)

Permutations

We will minimize our use of probability theories in Socy 601, but you might find the following section useful if you wish to go beyond the minimum expectations for the class.

If you have a set on *n* objects, you can arrange them in many orders. You can count the number of possible orders in the following way. There are *n* possibilities for your first pick. For your second pick there is one less possibility than for your first pick, because one object has already been picked. Your options keep decreasing, until you only have one possibility for your last pick – the last remaining object.

Example: what are the possible orders for the letters A, B, C, and D?
Solution: there are four possibilities for your first pick, which leaves three for the second, two for the third, and one for the last pick. The total possible number of orders is 4 * 3 * 2 * 1 = 24. (Try it yourself!)

Formal equation for number of sequences of *n* objects (out of a sample of *n* objects):

$${}_nP_n = n*(n-1)*(n-2)*\ldots*1 = n!$$

For smaller sets of objects, there are fewer permutations. (Example: how many ways can you pick *two* letters from the set of letters A, B, C, and D? Here is the formal equation when you want to pick only *r* objects out of a sample of *n* objects:

$${}_nP_r = n*(n-1)*(n-2)*\ldots*(n-r+1) = n! / (n-r)!$$

The above equations assume that the order is important – that is, that AB is different than BA. If you do not care about the order, but just want to know the number of possible groupings (combinations), the formal equation is as follows:

$${}_nC_r = n! / (n-r)! * (r)!$$

Example: how many combinations can you get when you pick two letters out of the set A, B, C, and D? (Don't replace the letters after you draw them.

Significant digits

When you write down calculations, always write at least three significant digits (digits that are not placeholders). I prefer that you do all computations in four or five significant digits to minimize rounding error. You can round your answers to fewer significant figures as necessary, but remember never to use a rounded answer in a subsequent calculation.

Sample numbers	Rounded to… Two significant digits	Three significant digits
5.66666666	5.7	5.67
145,586	150,000	146,000
0.951314127	1.0	0.951
0.00001003	0.000010	0.0000100

<u>Integral and Differential Calculus</u>

We use the ideas of calculus just a little bit when we talk about maximum likelihood estimation, but only for demonstration purposes. You are <u>not</u> required know calculus.

In the simplest terms, *differential calculus* includes techniques for finding the slope of a line at any given point on the line. *Integral calculus* includes techniques for finding the area under a line. These techniques are very useful when you are working with curved lines, and many important statistical ideas can be visualized as curved lines.